DOCUMENT RESUME

ED 443 423 IR 020 346

AUTHOR Forster, Patricia Ann; Mueller, Ute
TITLE Graphics Calculator Usage in the West Australian Tertiary Entrance Examination of Calculus.
PUB DATE 2000-04-00
NOTE 21p.; Paper presented at the Annual Meeting of the American Educational Research Association (AERA) (New Orleans, LA, April 24-28, 2000).
PUB TYPE Reports - Evaluative (142) -- Speeches/Meeting Papers (150)
EDRS PRICE MF01/PC01 Plus Postage.
DESCRIPTORS *Calculus; Comparative Analysis; Educational Theories; Foreign Countries; *Graphing Calculators; Graphs; High Schools; Higher Education; *Mathematics Instruction; *Mathematics Tests; Role; *Secondary School Mathematics; Social Theories; Tables (Data); Test Items; Use Studies
IDENTIFIERS Australia (Western Australia); *Technology Role; *Technology Utilization

ABSTRACT

This paper presents a comparative analysis of the Western Australian Calculus Tertiary Entrance Examination (TEE) papers for 1996-99, two years before and two years after the introduction of graphics calculators on the examination. Changes in the questions that asked students to graph rational functions and students' answers to the corresponding question from the 1999 paper are examined. Sociocultural theory and the use of technology are also discussed, including the role of the calculator, effects of technology usage, and inscriptions versus representations. The inquiry highlights implications for teaching and assessment of calculus in the presence of graphics calculators at the upper-secondary level. Results are presented in the following areas: (1) characteristics of questions for the 1996-99 Calculus TEE, including skills, real-life contexts, the role of diagrams, the role of graphics calculators, and effects on curriculum components; (2) questions on rational functions; (3) students' use of graphics calculators in the 1999 question; and (4) roles of the calculator. Contains 28 references. (MES)

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By Patricia Ann Forster & Ute Mueller

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P. Forster

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Graphics Calculator Usage in the West Australian Tertiary Entrance Examination of Calculus

Patricia Ann Forster, *Curtin University of Technology, Western Australia*
Ute Mueller, *Edith Cowan University, Western Australia*

In this paper we present a comparative analysis of the Western Australian Calculus Tertiary Entrance Examination papers for 1996-1999, two years before and two years after the introduction of graphics calculators on the examination. Then we take a closer look at changes in the questions that asked students to graph rational functions and finish with considering students' answers to the corresponding question from the 1999 paper. The inquiry highlights implications for teaching and assessment of calculus in the presence of graphics calculators at upper-secondary level.

Introduction

Since 1998 the availability of graphics calculators has been assumed for the West Australian public examinations of Mathematics, Physics and Chemistry at tertiary entrance (Year 12) level. This paper is part of a longitudinal study evaluating the impact of the introduction of the technology on the Calculus examination. Here we consider the characteristics of questions in the 1996-1999 Calculus papers, basing our analysis on a scheme that we developed from one by Senk, Beckmann and Thompson (1997). Then we look in detail at the changes in the style of questions that asked students to graph rational functions and finish by discussing the extent and nature of students' graphics calculator usage in the rational function question for 1999. Thus, we take a macro view of Calculus Tertiary Entrance Examination (TEE) papers for the two years before and the two years after the introduction of graphics calculators, then consider in particular the questions concerned with rational functions, to include a micro-analysis of students' answers to the 1999 question.

The analysis for the 1999 question on rational functions is based on written answers in examination scripts and on students' interview responses after the examination, describing how they used their graphics calculators. The epistemic referent for this part of inquiry was sociocultural theory, where descriptions of learning and assessment "are accounts of changing patterns of engagement in collective activities and social practices" (Renshaw, 1998, p. 84).

This paper provides a critical account of developments that have occurred in the assessment of calculus at high-school level, and adds to the research debate on the impacts of technology on mathematics education. Upon the introduction of graphics calculators into the Calculus TEE the role of diagram has increased, graphing questions have become less structured and the calculators have simplified the solution of some types of questions. However, testing in some curriculum components, for example complex numbers, has become higher level. Student use of graphics calculators in the rational function question under consideration here, from the 1999 Calculus TEE, varied from insightful through to unthinking acceptance of the outputs of the technology. Errors with graphing included students not noticing features that were displayed on calculator graphs and naive interpretation of the graphs, which did not take into account available algebraic and numeric information. The findings have implications for teaching and assessment.

Paper presented at the Annual Conference of the American Education Research Association, New Orleans, April 2000.

Background

The one-year course that prepares students for the Calculus TEE follows one-year of Introductory Calculus. The syllabus for Calculus assigns 6 hours to the 'Calculus of trigonometric functions'; 20 hours to 'Functions and limits'; 20 hours to 'Theory and techniques of calculus'; 28 hours to 'Applications of calculus'; 10 hours to 'Vector calculus'; and 21 hours to 'Complex numbers'. The emphases in the TEE papers on the various curriculum components reflect the time assigned to them in the syllabus. Each year approximately 1900 candidates from 140 schools participate in the examination, which is of three hours duration. The examination paper comprises about 20 questions worth a total of 180 marks and contributes equally with school assessment to students' tertiary entrance scores in Calculus.

Calculators without symbolic processing and the Hewlett Packard HP38G with limited symbolic capabilities are approved for examination purposes. This contrasts with the policy for the similar standard US Advanced Placement (AP) Calculus examination where the minimum level of capabilities assumed in setting questions is stated, but full symbolic capabilities are allowed (College Board, 1998). However, while the approach in AP Calculus is to require mathematical steps that lead to an answer, except that the setup only is required for definite integrals, equations and derivatives, a non-prescriptive approach has been taken to setting out required in the Calculus TEE. In addition, for the Calculus TEE, specific instructions to use graphics calculators are not generally given: students are expected to choose to use the technology when it is appropriate to do so. Because the text storage capacities differ between various brands of calculator, four A4 pages (two sheets) of notes are allowed.

Test and Examination Questions with Graphics Calculators

Characteristics of Examination Questions

Examination questions can be graphics calculator *active* where use of the technology is necessary or greatly simplifies a solution, graphics calculator *neutral* where graphics calculator usage and traditional methods are equally viable, or graphics calculator *inactive* where use of the technology is not possible (Harvey, 1992). A similar typology is suggested by Kemp, Kissane and Bradley (1996) where graphics calculators are *expected to be used*, *expected to be used by some students and not by others* and *not expected to be used*. When the graphics calculators were introduced for the Calculus TEE, the policy from the start was to include questions in all the above categories--there was no transition period where graphics calculators were optional equipment.

The presence of graphics calculators affects the selection of questions for examinations. Graphics calculators can *impact* on questions by enabling alternative methods, *have no impact* because they contribute no more than scientific calculators to a solution, or *trivialise* questions by allowing solutions that require little or no mathematical input from the user (Jones & McCrae, 1996). A solution can also be significantly *reduced in complexity* without the question being trivialised (see an example with complex numbers in Forster & Mueller, 1999). Questions can also be specially designed for technology usage, taking a different form to traditional questions and may include functions that students would not be expected to be able to manipulate by hand (Anderson, Bloom, Mueller & Pedler, 1997).

In an examination, graphics calculators can be used as the first option to generate answers or used to check non-graphics calculator methods (Jones & McCrae, 1996). Checking might involve replication of a hand method, or use of different representations, such as using a graph to verify algebraic working. Where a non-calculator approach is needed for the written answer, students might use the tool to get started (Lauten, Graham, & Ferrini-Mundy, 1994), to help with working and with the final answer. It is expected that procedural work is off-loaded to the technology, for example, for the evaluation of definite integrals (Jones, 1996).

Upon the inclusion of technology, the complexity and effectiveness of examination questions must be re-evaluated and changes must be made to modes of testing students' understanding of some concepts. Senk et al. (1997) developed a classification scheme to evaluate the nature of questions assuming the presence of technology and applied it in an analysis of the characteristics of test items used in 19 high-school classrooms. The inquiry included consideration of precalculus courses but did not extend to calculus. In general, test questions were found to be low level and either neutral or inactive with respect to the use of graphics calculators. Senk et al. concluded: "Clearly, classroom teachers, researchers, and test developers could all benefit from further discussion of how to recognise and write worth-while technology-active test items or other tasks" (p. 211). This paper contributes to that debate.

Sociocultural Theory and the Use of Technology

From a sociocultural perspective, mathematics is seen as an interactive activity, which is socially and culturally mediated. When tool-assisted, use of tools "is viewed as integral to mathematical activity rather than an external aid to internal cognitive processes located in the head" (Cobb & Bowers, 1999, p. 11). Cognitive activity is not considered as separate from calculator usage, but instead students are said to reason *with* the technology and there may also be residual cognitive effects (Salomon, Perkins & Globerson, 1991). Literature on interactive relationships between students and technology, on effects of technology and on multiple representations guided our inquiry into students' reasoning with graphics calculators and is briefly reviewed below.

Role of the Calculator

Galbraith, Renshaw, Goos and Geiger (1999) distinguish between four different interactive roles of graphics calculators. They might be like *servants*, obediently carrying out graphing and calculation. However, with student acceptance of information on a screen display, irrespective of its accuracy, the role of the technology changes to *master* (Galbraith et al.). Here students' involvement might be classified as *mindless*, "characterised by blind reliance on marked structural features of a situation without attention to its unique and novel features" (Salomon et al., 1991, p. 4). This contrasts with *mindful engagement*. A student might also work in *partnership* with technology (Galbraith et al.; Jones, 1996; Salomon et al.): here the calculator "becomes a friend to go exploring with . . . where the output needs to be checked against known mathematical properties" (Galbraith et al., p. 225). Technology can also be an *extension of self*, where "the partnership between student and technology merges into a single identity . . . [resulting in] an extension of the user's mathematical prowess" (Galbraith et al., p. 225).

These interactive relationships vary in the degree of control exerted by students in their

technology usage, from high levels, for example in a partnership, to low levels, where information is accepted as though from a master. Another aspect of control is that the design of technology to some extent determines or controls how problems are solved (Salomon & Perkins, 1998). For graphics calculators, this is evidenced in the way they allow direct entry and manipulation of algebraic and numeric forms but not of graphical forms (Kaput, 1998). A graph has to be converted to its algebraic or numeric equivalent before the calculator can be used.

Effects of Technology Usage

Working with technology may lead to the ability to solve problems becoming *distributed* between the user and the tool (Pea, cited in Salomon et al., 1991), and this might result in de-skilling in some areas but enhancement of students' performance in others. Salomon et al., again drawing on the work of Pea, describe two benefits of technology: effects on ability when working *with* technology and cognitive residue effects *of* technology. Berger (1998), also referring to Pea, elaborates on these benefits, using the terms *amplification* and *cognitive re-organisation*. The graphics calculator *amplifies* the zone of proximinal development by carrying out processing tasks so that the child can achieve more with the technology than he/she could achieve without it (Renshaw, 1998). The cognitive re-organisation effect is a systemic change in the consciousness of the learner as a result of using technology (Berger, 1998). However, evidence of the cognitive reorganisation effect can be elusive: in a study of first-year university calculus students Berger found that students with graphics calculators did not, except for one instance, approach diagnostic problems in qualitatively different ways to students without the technology. She attributed this to the calculator being an add-on tool, and to the *privileging* of traditional methods. Kaput (1998) describes privileging in terms of a deep cultural bias so that "what counts as significant mathematics is typically taken to be mathematics expressed in terms of character strings" (p. 275).

Inscriptions Versus Representations

Kaput (1998) distinguishes between *representations*--marks (signs and symbols) in a structured system standing for something else, and *inscriptions*--marks "in a physical medium apart from any reference to how they might be used, understood, or perceived, and, apart from any structure they might embody, from the third-party point of view" (p. 270). From this viewpoint, a graphical display on a calculator has no meaning in itself, but its significance arises from the social and cultural context in which it is produced and used: Roth and Bowen (1998) describe how experts interpret graphs by "[c]ycling back and forth between actual situations they are familiar with (the referent) and the graph (the sign)" (p. 5). Kaput suggests that mathematical symbolism as representation fits a cognitivist perspective and as inscription fits a sociocultural view.

Roth and McGinn (1998) cite graphs, tables and equations as examples of inscriptions and, as such, properties that are highly relevant to graphics calculator usage include that: "Inscriptions are easily rescaled to produce larger or smaller images without changing their internal relations . . . are easily combined and superimposed . . . can be reproduced . . . are often translated to other inscriptions" (pp. 37-38). The reproduction of the graphical display on a calculator on paper is documented as a source of error for students. Errors might be attributable to *misreading* the actual display so that, for instance, asymptotic behavior is not recognised (Boers & Jones, 1994; Mueller & Forster, 1999; Ward, 1997). A curve might copied without its curvature being made apparent because it resembles a straight line on the

calculator due to the scale chosen (Mueller & Forster, 1999)--*a scaling error* (Roth & McGinn, 1998). A display might be interpreted literally so, for instance, the maximum absolute value might be read from the maximum turning point even though the minimum turning point value has greater magnitude (Forster & Mueller, 2000)--*an iconic error* (Roth & McGinn, 1998).

Research Method

Comparative Analysis of the 1996-1999 Calculus TEE papers

We decided to restrict our analysis of the Calculus TEE to the 1996-1999 papers on the basis that in 1996 the format of the papers changed to having questions ordered according to their degree of difficulty. Previously, the papers contained two sections. One with routine questions without interdependent parts and the other with longer questions, typically more demanding with interdependent parts. Of the four papers we consider, the 1996 and 1997 papers were set before and 1998 and 1999 papers after the introduction of graphics calculators.

For the analysis we used a coding scheme that we modified from one by Senk et al. (1997). First, we independently coded question characteristics according to the original scheme of Senk et al., then modified the scheme to suit the Calculus TEE (see Table 1 next page). Finally, we independently recoded the questions and, where we varied, we negotiated agreement guided by the official worked solutions.

The role of a diagram depends to a large extent on the ease with which a diagram may be obtained. Graphs can be quickly generated on graphics calculators, whereas without them drawing a graph might be impractical. When coding for 'Role of diagram', we took into account the absence of graphics calculators for 1996 and 1997 and their presence for 1998 and 1999. For interest, we coded 'Active', 'Neutral' and 'Inactive' for 'Graphics Calculator', whether the technology was available to be used (1998 and 1999) or not (1996 and 1997). Some questions belonged to more than one curriculum component and were recorded as belonging to each group.

Questions on Rational Functions and their Graphs

Coding the papers drew our attention to distinct changes over the four years for questions concerned with graphing, rectilinear motion, simple harmonic motion and complex numbers. Analysis of the quantitative data indicated a high level of calculator usage in 1998 and 1999 for questions which asked students to graph rational functions and this was further reflected in students' interview responses. Together, these factors influenced us to focus on the rational function items in this paper.

Four types of data were collected for the larger inquiry of which this paper is part. First, three students from each of two schools and four students from another were interviewed about calculator processes they used in the examination. Students were selected by their teachers on the basis of being communicative. Their school assessment grades ranged from A to C (D is the lowest pass grade). The examination paper was used as a heuristic in the interviews. The second type of data was results for all candidates (marks per question), obtained from the Curriculum Council of Western Australia who administer the TEE examinations. Third, examination markers recorded data on a proforma, for a sample of their allocated scripts. Data were generated for seven questions, including parts (b) and (c) of the

Table 1
Categories for Analysing Examination Questions

Category	Description
Curriculum component	Functions and Limits Theory and Techniques of Calculus Applications of Calculus Vector Calculus Complex Numbers
Skill	
Yes	Solution requires a well-known algorithm such as solving equations or inequalities or bisecting an angle. Item does not require translation between representations
No	No algorithm is generally taught for answering such questions, or item requires translation across representations
Level	
Low	A typical student in that course would use no more than three steps to solve.
Other	A typical student in that course would use four or more steps to solve.
Reasoning	
Yes	Item requires justification, explanation or proof or it is necessary to interpret the question before being able to start the answer.
No	No justification, explanation or proof is required. (By itself, 'Show your work is not considered reasoning.)
Realistic context	
Yes	The item is set in a context outside of mathematics (e.g. art, fantasy, science, sports).
No	There is no context outside mathematics.
Role of diagram	
Interpret	A graph or diagram is given and must be interpreted to answer the question.
Make	From some non-graphical representation (data, equation, verbal description) the student must make a graph or diagram.
Assist	The use of a diagram or sketch would simplify a solution, but is not essential for obtaining the answer.
None	No graphical representation is given or needed or a graph or diagram is given but is superfluous to answering the question.
[a]Graphics Calculator	
Active	Use of the tool is necessary to obtain a solution or it greatly simplifies the work needed to get a solution.
Neutral	It is possible to use the tool to obtain part or all the solution, but the question could be answered reasonably without the tool
Inactive	Use of the tool is not possible or is inappropriate.

[a] over and above scientific calculator capabilities

three-part question asking students to graph a rational function. We decided not to consider part (a) as it would have been difficult to deduce whether the calculators had been used or not to answer it. Part marks were recorded and columns ticked to indicate if students' methods were traditional or graphics calculator based; and ticked to indicate specific graphical features in students' answers. Nine out of 24 markers volunteered to be involved. All were experienced teachers and this resulted in data for 195 scripts. The fourth type of data consists of our own observations, in our role as examiners and markers. We recorded data while marking 240 scripts (from the total 1957 scripts--scripts are marked twice) to obtain a total sample of size 435.

The sample statistics are only a guide and not a definitive statement of graphics calculator usage--discerning graphics calculator usage from examination scripts is necessarily interpretative and the sample was not randomly selected. However, scripts from a school are distributed between the bundles for marking, and bundles are allocated to markers without preference. In order to assess the quality of our sample we compared the population marks distribution for part (c) of the 1999 question on rational functions with the mark distribution of our sample. This was the only part for which we had both sample and population data. A chi-squared goodness of fit test on students' scores shows that the sample distribution is representative of the population distribution. The χ^2 value is 3.76 and there are 6 degrees of freedom.

Results

Characteristics of Questions for the 1996-1999 Calculus TEE

A summary of our comparative analysis for all questions on the 1996-1999 papers is given in Table 2. Changes in the nature of examinations can be attributed to a variety of factors but possible influences include the following.

Skills. For all four examination papers the majority of the questions were skills-based (see Tables 2 and 3). This pattern has not changed to any large extent upon the introduction of graphics calculators, but scrutiny of the questions showed that the skills that are tested have changed. For example, the question, 'Evaluate $(1+i)^7 + (1-i)^7$ using de Moivre's rule, from the 1996 examination has been made redundant by the presence of graphics calculators. The use of de Moivre's rule used to be essential for obtaining the answer to expressions of this type quickly--the other alternative of using the binomial theorem would have been much more time-consuming. Thus the ability to apply de Moivre's rule correctly was an essential skill. With a graphics calculator the simplification requires only a single line entry of the character string. Here, requiring students to use de Moivre's rule is inappropriate, just like asking students to use a calculator to find logarithms in order to solve an exponential problem (Jones, 1996).

A further change in the papers is that in 1996 and 1997 testing of integration techniques was based on the evaluation of definite integrals while now indefinite integrals are used exclusively because of the numerical integration capabilities of graphics calculators. Definite integrals have been incorporated to a greater extent in application questions. Similarly, questions that require factorisation of polynomials as the sole task are now absent in view of them being *trivialised*.

Table 2

Percentage of Part-questions in the Calculus Tertiary Entrance Examinations for 1996-1999 per [a] Major Curriculum Component by Characteristic

		Skill-based	Level	Reasoning	Context	Role of Diagram				Technology		
		No	Other	Yes	Yes	Interpret	Make	Assist	None	Active	Neutral	Inactive
Function and limits	1996	21	43	29	0	7	7	0	86	14	57	29
	1997	13	20	27	13	0	7	13	80	7	87	7
	1998	20	40	70	0	0	10	60	30	10	50	40
	1999	15	8	31	0	15	7	46	38	15	54	31
Theory and techniques of calculus	1996	27	45	27	0	0	0	27	73	27	9	64
	1997	0	23	46	8	0	0	8	92	23	15	62
	1998	10	40	20	10	0	0	0	100	10	0	90
	1999	20	40	30	0	0	0	0	100	10	0	90
Applications of calculus	1996	38	77	69	15	8	8	15	69	31	31	38
	1997	25	75	83	33	0	8	33	58	25	17	58
	1998	44	61	67	56	33	11	11	45	17	6	78
	1999	40	40	45	60	14	14	19	52	30	10	60
Complex numbers	1996	43	43	36	0	14	29	0	57	7	36	57
	1997	63	38	38	0	38	0	0	63	13	38	50
	1998	56	56	44	0	0	44	22	33	22	0	78
	1999	80	60	60	0	0	50	20	30	0	30	70
[b] % of total part- questions	1996	30	46	37	13	7	11	9	72	17	33	50
	1997	20	41	45	14	6	4	14	76	18	39	43
	1998	33	53	55	27	12	14	20	53	14	12	73
	1999	35	35	40	28	10	14	21	55	16	23	61

[a] Calculus of trigonometric functions (6hours of coursework) and Vector calculus (10hours) are not included as few part questions fall only into these categories

[b] number of part-questions were 54 for 1996, 51 for 1997, 49 for 1998, 57 for 1999

Real-life Contexts. Questions set in real-life contexts doubled from 1996-1997 to 1998-1999 (see Table 2). This is partly attributable to preferences of examiners but also reflects a move away from procedural towards more interpretative, applied questions where graphics calculator usage is an option.

Role of Diagram. Diagrams now play a greater role in problem solutions (see Tables 2 and 3). There are more part-questions that require interpretation of a diagram or where students are required to make a diagram, and more part-questions where a diagram would assist the solution. This enhanced role for diagram may be attributed in part by the relative ease with which students can obtain graphs of functions, parametric and polar curves and in part by the ease with which markers can generate graphs to 'follow through' students' solutions. In 1999 a question asked students to derive and graph the velocity function from the position $s(t) = (t^2 + 1)/(t^4 + 1)$. The derivation is potentially problematic, as is graphing the velocity function with traditional methods. The task of marking the question would have been arduous without having the technology available to check answers that followed-on from the velocity functions students obtained--without the technology available the above position function would not have been used in a question of this type.

Role of graphics calculators. Overall, opportunities to use graphics calculators, had they always been available, have not increased with the introduction of the technology for the TEE: see Table 2, which gives the breakdown in usage according to part-questions. On the basis of part-marks, in 1996 and 1997 approximately 53% of all marks could have been obtained through graphics calculator active or neutral part-questions, this percentage was 26% in 1998 and 39% in 1999. The lower mark allocation is largely caused by omission of skills-based questions of the type included in 1996 and 1997 that would be trivial in the presence of graphics calculators.

Effects on Curriculum Components. The effects described above have impacted on the various curriculum components in differing ways. The summary in Tables 2 suggests that diagrams, usually graphics calculator generated graphs, could have assisted in answering questions from the component 'Functions and limits' more in 1999 than previously. This enhanced role of diagram went hand-in hand with a reduction of the amount of guidance given for graphing, and we explore this in more detail in the next section in regard to rational functions. Another aspect of students being able to graph readily was the use of more complicated functions. For example, in the 1999 examination the function $f(t) = \begin{cases} (1-\cos(2t))/t & \text{for } t \neq 0 \\ 0 & \text{for } t = 0 \end{cases}$ was used to test understanding of limits, continuity and other properties of functions. Without access to a graphics calculator, either the graph would have been supplied or graphing would have dominated the question. Otherwise the requirement for a graph would have been omitted in favour of algebraic methods, thereby making the question too abstract to be a suitable examination question at this level. In its present form, the question allowed students the opportunity to demonstrate their mathematical insight with an unfamiliar function.

Another change that became evident through the comparative analysis was that questions for 'Theory and techniques of calculus' (integration and differentiation techniques) now largely preclude graphics calculator usage (see Table 2). For 'Applications of calculus', there is no pattern of increased opportunity for calculator usage (see Table 2), but in 1998 and 1999 it is the component for which there were opportunities for flexible problem solving (Gray & Tall,

1991) with both traditional and technology-assisted methods equally viable. For example, graphical or trigonometric methods could have been used in a 1999 question where tidal fluctuations were modeled as simple harmonic motion (Forster & Mueller, 2000).

The component most affected by the introduction of the calculators (see Table 2) is 'Complex numbers'. Questions have become less skills-based, need a greater number of steps to reach an answer, and call on more reasoning. That is, in general in 1998 and 1999 the questions were harder than those for 1996 and 1997 in all the dimensions that measure difficulty. Diagrams played a greater role, but usually these were not graphics-calculator generated--there was actually reduced opportunity to use graphics calculators in questions on the topic.

Section Summary. With the inclusion of graphics calculators in the Calculus TEE, skills tested for some syllabus items have changed and, consistent with recommendations in current reform documents (e.g., Curriculum Council, 1998, NCTM, 1995), there are increased numbers of questions set in real-life contexts. There is an increased role for diagram in general, but particularly in the 'Function and limits', 'Applications of calculus' and 'Complex number' components of the curriculum.

Questions on Rational Functions

Here we discuss in more detail the changes in style of questions on graphs of rational functions. We focus on the 1996 question that was answered without the technology and the 1999 question where technology was available and used for the graph by a large majority (85%) of students, judging from the sample of 435 scripts (based on little working being shown). The codings for the 1996 and 1999 questions are provided in Table 3.

1996. Question 10. Let f be the function defined by $f(x) = \dfrac{-4x+5}{(x+2)(5-x)}$.

(a) State the poles of the function.
(b) Evaluate $\lim_{x \to +\infty} f(x)$.
(c) Evaluate $\lim_{x \to -\infty} f(x)$.
(d) Evaluate $\lim_{x \to 4} f(x)$.
(e) State the x and y intercepts.
(f) Show that there are no turning points and sketch the graph of $y = f(x)$, clearly labeling all the important features.

1997. Question 17. If $\dfrac{2x^2 - x}{x^3 - 1}$,

(a) State the pole of the function.
(b) Evaluate $\lim_{x \to \infty} f(x)$.
(c) Evaluate $\lim_{x \to -\infty} f(x)$.
(d) State the x and y intercepts.
(e) The derivative is given by $f'(x) = \dfrac{-2x^4 + 2x^3 - 4x + 1}{(x^3 -)^2}$. The numerator of $f'(x)$ has exactly two roots. One of these roots is located at approximately 0.26, the other is near -1. Use the Newton-Raphson method to refine the root near -1 to two decimal places.
(f) Classify the critical points as a local maximum, minimum or points of inflection.

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(g) Sketch the graph of f. Clearly label all the important features.

1998. Question 14. If $f(x) = 1 - \dfrac{x}{(x-1)^2}$,

(a) Sketch the graph of $f(x)$, indicating all asymptotes and turning points.
(b) For each of the following initial values decide whether the Newton-Raphson method would lead to the left root, the right root or neither.
(i) $x_0 = 2$ (ii) $x_0 = -2$ (iii) $x_0 = 1/4$

1999. Question 13. If $f(x) = \dfrac{x^2 + 3x - 10}{x^2 + x - 6}$,

(a) state the domain of f.
(b) evaluate $\lim_{x \to 2} f(x)$,
(c) sketch the graph of f showing the intercepts, asymptotes and any other distinguishing features.

Table 3
Coding for the 1996 and 1999 Calculus TEE Questions on Graphing Rational Functions

	Skills-based	Level	Reasoning required	[a]Role of diagram	[b]Graphics Calculator
1996					
a	Yes	Low	No	None	Inactive
b	Yes	Low	No	None	Neutral
c	Yes	Low	No	None	Neutral
d	Yes	Low	No	None	Neutral
e	Yes	Low	No	None	Neutral
f	No	Other	Yes	Make	Active
1999					
a	Yes	Low	No	None	Neutral
b	Yes	Low	No	Assist	Neutral
c	No	Other	Yes	Make	Active

[a] no technology available for 1996
[b] answered as though technology was available for 1996

The 1996 question (see above) set prior to the introduction of graphics calculators is very highly structured. Part questions, which relied on recognition of properties pertaining to the function and involved algebraic manipulation, led students item by item through features of the graph, attracted part marks and the tasks were low-level and solutions were skills-based (see Table 1 and 3). The final step of drawing the graph was higher level (see Table 1 and 3) and required students to link together a number of mathematical properties in order to draw the graph. However, once the graph was drawn no further interpretation was required. The process was essentially linear: interpretation of the question → algebraic manipulation → hand draw the graph.

In 1999, two part questions led students into the graph and students were warned to mark 'other distinguishing features'. Choosing to use a graphics calculator meant the process of answering the question started similarly to above: interpretation of the question → algebraic manipulation. But then, after using the calculator for graphing, students needed to interpret the calculator graph in the light of properties previously established, rather than using the properties to plot the graph. For the identification of all the graphical features, students needed to integrate (Boers & Jones, 1994) their reading of the calculator graph with information in

the question and with mathematical properties established in the previous two part-questions.

The 1997 question was similar to that from 1996 in that the initial parts (a) to (f) led students into the graph, which they drew in the absence of technology. Part (f) which relied on part (e) was relatively demanding (see the question on the previous page). The 1998 question asked students to draw the graph, with no lead in parts. It attracted widespread calculator usage (Mueller & Forster, 1999) where again, as for the calculator-assisted graphing in 1999, a process of integration (of information in the question with the calculator graph) was involved. A second, interpretative part followed. The mean scores for the rational function questions from the four years given in Table 4.

Table 4
Results for the Population for the 1996-1999 Questions on Graphing Rational Functions

	1996	1997	1998	1999
Mean % mark for the question	82%	72%	74%	78%
Total marks available for the question	14	21	8	10

Overall the results in Table 4 do not indicate that students found the rational function questions easier or harder upon the availability of graphics calculators, even with the different processes involved. However the introduction of the technology has been accompanied by a lower mark allocation to the questions, explained by fewer or no lead-in parts to the graph, that is fewer marks allocated to algebraic manipulation that has traditionally been a dominant feature of graphing questions.

Students' Use of Graphics Calculators in the 1999 Question

The question in the 1999 Calculus TEE asking students to graph a rational function was the thirteenth of the twenty questions, of which all were to be answered. First we provide the question again, for convenience of reading, then students' descriptions of methods that they used for each part of it. Then we give summary tables of data generated from the sample of scripts. Thus, initially the description is of the extent and nature of students' graphics calculator usage, including errors made. This section is followed by a discussion on the roles that graphics calculators played in students' answering of the question.

Question 13. If $f(x) = \dfrac{x^2 + 3x - 10}{x^2 + x - 6}$, (a) state the domain of f, (b) evaluate $\lim_{x \to \infty} f(x)$, (c) sketch the graph of f showing the intercepts, asymptotes and any other distinguishing features.

Students adopted a variety of traditional and graphics calculator approaches to each part of the question, and for part (a), which asked for the domain, these included:

[1] I looked at the denominator [and worked it out by inspection].
[2] I factorised it. If I'd graphed it and hadn't done this out, I wouldn't have got all values . . . the zeroes in the denominator. [$(x^2 + x - 6) = (x - 2)(x + 3)$, domain: all reals $\neq 2$ or -3]

For part (b), which asked for the limit as $x \to 2$, one strategy students adopted was to use their graphics calculators for calculation:

[3] I substituted values . . . in run mode.
[4] I put the whole function in 'Function'. Then I went into 'Num' (see Figure 1) and put in a number less than 2 and a bit more than 2.

Figure 1. Graphics calculator table of values for $f(x) = (x^2 + 3x - 10) / (x^2 + x - 6)$

Others use a graphical approach:

[5] I graphed it and found a break in it. I did the positive side and the negative side (Figure 2).

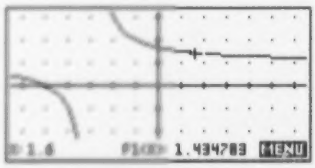


Figure 2. Graphics calculator graph of $f(x) = (x^2 + 3x - 10) / (x^2 + x - 6)$

With a traditional (non-graphics calculator) method, students mainly used factorisation but also L'Hopital's rule:

[6] I did the limit of the derivatives . . . L'Hopital's rule, then checked it in 'Num' (see Figure 1).
[7] I prefer doing it manually. That's why I didn't like the paper so much because the paper relied on the calculator.

Table 5 summarises the incidence of graphics calculator and traditional methods for the limit.

Table 5
Methods Students Adopted for a Limit as Deduced from Written Answers

	13(b)	
	traditional	graphics calculator
*No. students choosing the method	223 (58%)	160 (42%)
Mean mark out of 2	1.8 (90%)	1.9 (93%)

* percentages are for students who are recorded as answering the question from the sample of n=435

For part (b), calculator usage was deduced from the provision of a table of values or a graph, or from lack of working and 42% of those who answered the question appeared to use their calculators (see Table 5). The results showed that there was no substantial difference in the average scores for the two approaches.

Part (c) asked students to graph the function *f*. Features of the graph included vertical and horizontal asymptotes and a point discontinuity. Intercepts also had to be identified. Table 6 is a summary of students' scores and the extent of calculator usage for generating the graph, as indicated by minimal or no working in the sample of scripts. The majority of students (85%) seemed to rely on the calculator option (see Table 6). Again the difference in marks for the

two alternatives was not substantial.

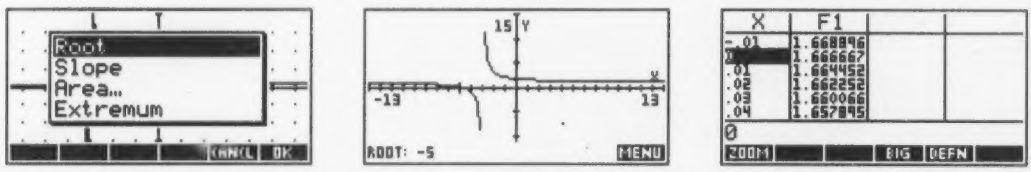
Table 6
Methods Students Adopted for Generating a Graph as Deduced from Written Answers

	13(c)	
	traditional	graphics calculator
[a]No. students choosing the method	63 (15%)	363 (85%)
Mean mark out of 6	4.4 (73%)	4.5 (74%)

[a] percentages are for students who are recorded as answering the question from the sample of n=435.

Strategies adopted on the calculators to identify the features of the graph included:

[8] I found the intercept [on the horizontal axis] using 'Root' (see Figure 3A, 3B), then went into 'Num' and put in zero to get the *y* intercept (Figure 3C).



A B C

Figure 3. Calculating the root (A and B) and *y* intercept (C) of $f(x) = (x^2 + 3x - 10) / (x^2 + x - 6)$

For the horizontal asymptote:

[9] I could visually see it.
[10] It was at $y = 1$. I just looked at the graph and saw it was going to '1' and put a really big number into 'Numeric'.
[11] I went into 'Num' and put in big numbers . . . positive and negative numbers.
[12] I evaluated it in my head. I divided by the highest order and put in infinity. Then I checked on the graph again. It looked as though it approached from one below on the left and one below from the right. It was worth taking the extra time because it [the graph] was worth six marks.

The position of horizontal asymptote could be deduced from the graphical display if the scales were set appropriately and this was potentially assisted by the selection of a grid for the display (see Figure 2). In regard to the vertical asymptote $x = -3$:

[13] There was a pole at $x = -3$. . . I could see it on the graph.
[14] It was the one [from part (a)] on the negative side.

In students' written answers there were isolated instances of the branches of the curve terminating on the vertical asymptote rather than the branches approaching it, which is attributable to literal reading of the calculator display (see Figure 3B): an iconic error which arises from discretisation of the co-ordinate plane on a calculator. As to the point discontinuity at $x = 2$, it was visible on some calculator graphs (see Figure 2). In referring to it, students commented:

[15] It was indeterminate at $x = 2$ so I put an empty circle joining the curve.
[16] I marked it with a break.

However, the point discontinuity was often missed. The discontinuity is only visible on a graphics calculator if it coincides with a pixel on the screen and this is dependent on choosing appropriate scales and screen resolution (where this is an option). Even when the discontinuity appears on the screen, it is usually minute and is easily missed if relying on visual methods. Factorising the numerator and denominator of the rational function, then canceling the common factor results in $(x+5)/(x+3)$ and entering it in the calculator results in a curve without a discontinuity at $x=2$. Interview comments indicate the confusion some students experienced in regard to the discontinuity:

[17] I wasn't really sure because it doesn't really exist at $x = 2$, so I put an open circle.
[18] It wasn't an asymptote, just a discontinuity so I just put an empty circle. It looked a bit funny, so I checked my equation about three times [that it was correctly entered], then looked at 'Num' which showed undefined at 2.
[19] I went into equation to find the zeroes on the bottom [-3 and 2] . . . -3 was the only asymptote [the student didn't identify the point discontinuity].

There were isolated instances of students drawing a graph with two asymptotes, at $x=-3$ and 2. Here students could be seen to ignore the information from part (b) and to favor, over a calculator graph, the values excluded from the domain that had been established in part (a). The extent to which asymptotes and the point discontinuity were included in written answers, for the sample of scripts is summarised in Table 7.

Table 7
Inclusion of Asymptotes and a Discontinuity on the Graph of the Rational Function

	Horiz. Asympt.	Vert. Asympt.	Pt. Discontinuity
Traditional approach (n = 63)	50(80%)	61(97%)	25(40%)
Calculator approach (n = 363)	272(75%)	352(97%)	183(50%)

Roles of the Calculator

Student comments portray how they worked *with* their graphics calculators. First, students could be said to have used their calculators as one might a robot or a *servant* (Galbraith et al., 1999) to automate numerical calculation (e.g., line 3), to produce a graph (e.g., line 5), and to carry out calculation associated with a graph (e.g., line 8). Here the technology was "a reliable timesaving device for mental, or pen and paper computations . . . the output is regarded as authoritative, although the discerning user will continue to monitor reasonableness" (Galbraith et al., p. 255). In addition to monitoring the viability of calculator outputs, with calculator graphs students need to check their interpretation of them. We saw this checking in a student switching from a graph to a table of values (line 10). However, it seemed that for part (c) some students accepted without question the calculator graph and their interpretation of it as reliable (e.g., line 9). This lack of checking is one explanation for why one quarter of students in the sample omitted the horizontal asymptote on their hand drawn graphs, and half the students omitted the point discontinuity (see Table 7).

There was also evidence of higher-level usage of the technology. Some students, judging by their interview responses, seemed to work in *partnership* with their graphics calculators, to verify an algebraic method against a calculator table of values (line 6), check a mental calculation against a calculator graph (line 12) and confirm the nature of a discontinuity in the

table of values (line 18): "a feature of this [partnership] mode is the way in which the respective 'authorities' of mathematics and technology are balanced" (Galbraith et al., 1999, p. 225). In two of these instances (lines 12 and 18) student endeavor resembled a cyclic process--where students *integrated* (Boers & Jones, 1994) their interpretation of the graph with numeric outputs and algebraic information. Like experts in the field, they moved back and forth between referents with which they were familiar and the graphs they were trying to interpret (Roth & Bowen, 1998). However, here the referents were information available on the calculator and in the question, rather than real phenomena to which that Roth and Bowen refer. The students' efforts are indicative of *mindful engagement* (Salomon et al., 1991), where they intelligently interacted with their calculators. However, among the population of candidates (N = 1937) only 598 students, about one third of the cohort, achieved full marks for the graph. This suggests that in the vicinity of two thirds of students failed in one or more respects to move to high-level calculator use, characterised as working in partnership with the technology. In some instances, students perhaps were aware that the calculator output was inconsistent with properties they had already established but were not able to resolve their confusion, and this is illustrated in lines 17 and 19. One student opted for what she expected from early parts of the question (line 17) to successfully identify the point discontinuity; the other copied the calculator graph which did not show the discontinuity so was acting in subservience to the technology (line 19). The *technology had mastery* over him (Galbraith et al., 1999).

As to the effects of technology discussed in the literature, we note that there was limited opportunity in the question for students to benefit from the amplification effect: there were no tasks further to the graphing where students could have been advantaged through having had the calculator generate the graph. In the Calculus TEE, such further tasks are generally reserved for applied questions. The second effect is cognitive reorganisation or thinking differently as a result of technology usage. Traditionally, domains and limits of functions were established algebraically. In the presence of technology, some students took a graphical approach to finding the limit in part (b) of the question, as indicated by the student comment (line 5) and by the lack of working in written solutions. This may be regarded as an instance of cognitive reorganisation. However, other students privileged traditional methods, choosing manual approaches almost exclusively (line 7) or using the technology as a backup rather than the first option (line 6). This privileging was reflected in over half the sample of 435 students using traditional methods for evaluating the limit in part (b) (see Table 1); and was desirable for determination of the domain. As a student mentioned (line 2), undefined values in a domain can be difficult to detect if relying solely on the calculator and consequently the point discontinuity might be missed. As Dreyfus (1994) suggests, we should aim for a balance between algebraic and visual methods, and more than that, we should aim for students to integrate them.

Conclusion

The comparative analysis of the 1996-1999 Calculus TEE papers has made explicit changes in the examination questions that have accompanied the introduction of graphics calculators. The inquiry has shown that the availability of technology has impacted on the way concepts are tested and on what skills are tested. The use of de Moivre's rule to evaluate a complex number raised to a given power has become redundant, and polynomial factorisation questions have become trivialised. Definite integrals are no longer suitable for testing integration techniques. Yet, the concepts associated with de Moivre's rule, polynomials and

definite integrals remain important aspects of calculus. There is a need to rethink how to test students' understanding of them, and in the Calculus TEE this involved a greater role for diagrams for questions on complex numbers. An increased role for visual methods is a notable change for the papers overall. The implications for teaching are that the introduction of graphics calculators has reduced the importance of some skills but this does not apply to the concepts to which they relate, and other skills associated with graphical interpretation seem to warrant more emphasis.

Casting the calculator in the role of servant, partner or master drew our attention to ways in which students worked with the technology for the question on graphing a rational function. The inquiry highlighted what seemed to be relatively uninformed calculator usage by the majority of students. This was reflected in written answers where students seemed to have directed their calculator as one might a servant, but then merely copied the calculator graph without exploring all or any finer details, resulting in widespread omission of asymptotes and a point discontinuity, and other isolated errors. As Boers and Jones (1994) observe, critical awareness of the limitations of calculator graphs and ability to integrate calculator outputs with other information do not come automatically with ownership of the technology. The implication for teaching is that these competencies need to be made explicit and nurtured.

The policy for the Calculus TEE is that graphics calculator usage is expected--it is part of the culture, although would have been embraced more into the curriculum by some teachers than others. Notwithstanding, we might expect more evidence as to its cognitive effects than was noted by Berger (1998) in a study where the technology was an add-on tool. There was evidence of cognitive reorganisation--students adopted graphical approaches instead of traditional ones and this was both a benefit (for a limit to infinity) and a hindrance (to identifying point discontinuities). Again, the implication for teaching is that these benefits and difficulties need to be subjects of instruction.

An important aspect of the inquiry for us, which has implications for assessment in general, was to gain a better understanding of how to test students' abilities to work with technology. The question concerned with graphing for 1999 did not test conceptual reasoning, other than requiring the interpretation of a graph. It did not have an interpretative part further to the graphing where students could show that they could achieve more with the calculator than without it. The inclusion of such part questions seems appropriate, contingent on them meeting the overall purpose of the Calculus TEE (or other examinations like it) to test students' understanding of calculus, keeping in mind that one criterion is to have a balance of 'easy' and 'difficult' questions.

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